Introduction to "Standard" Flux-Rope Fitting

B. J. Lynch[1,2], T. H. Zurbuchen[1], S. K. Antiochos[2,1]

1. Department of Atmospheric, Oceanic, and Space Sciences, University of Michigan, Ann Arbor, MI, 48105
2. E. O. Hulburt Center for Space Science, Naval Research Laboratory, Washington, DC, 20375

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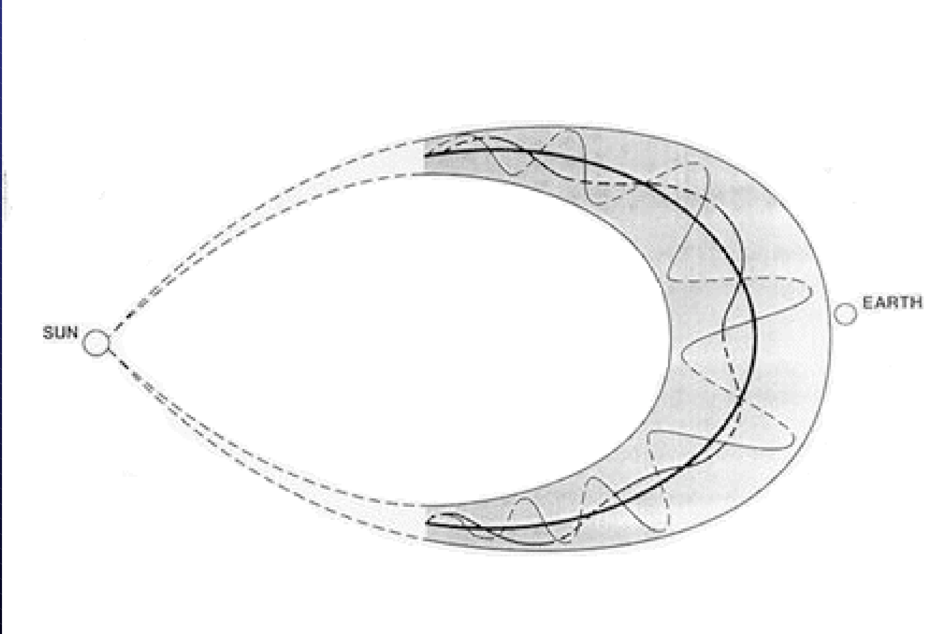
Outline

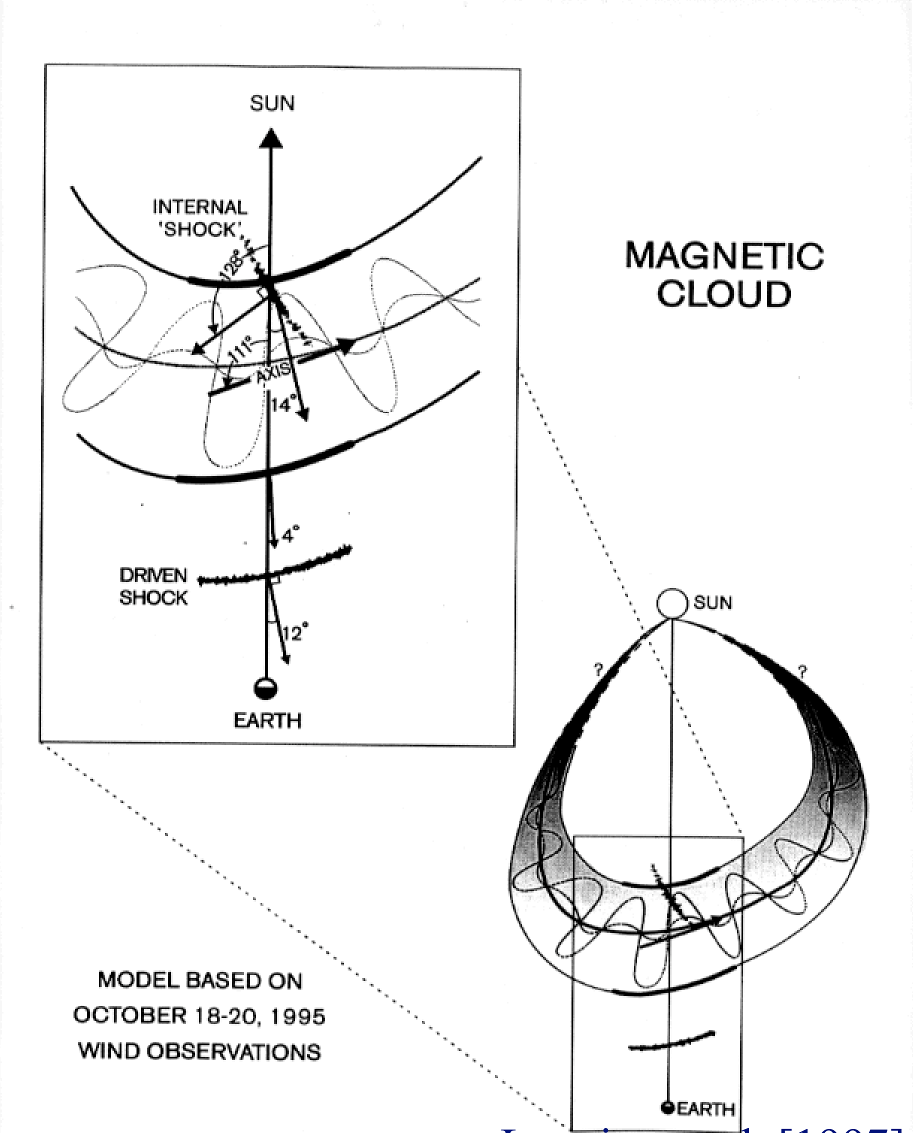
- Motivation
- Lundquist Solution + Parameters
- Fitting Procedure
- Example Fits (Recycled)
- Fitting Uncertainties
- Conclusions

Motivation: Interplanetary Structure

- Interpretation of large-scale, coherent magnetically dominated plasma structures (driver gas)!
- Implications for propagation, background heliospheric structure
- Multi-spacecraft measurements sometimes consistent with cartoon!



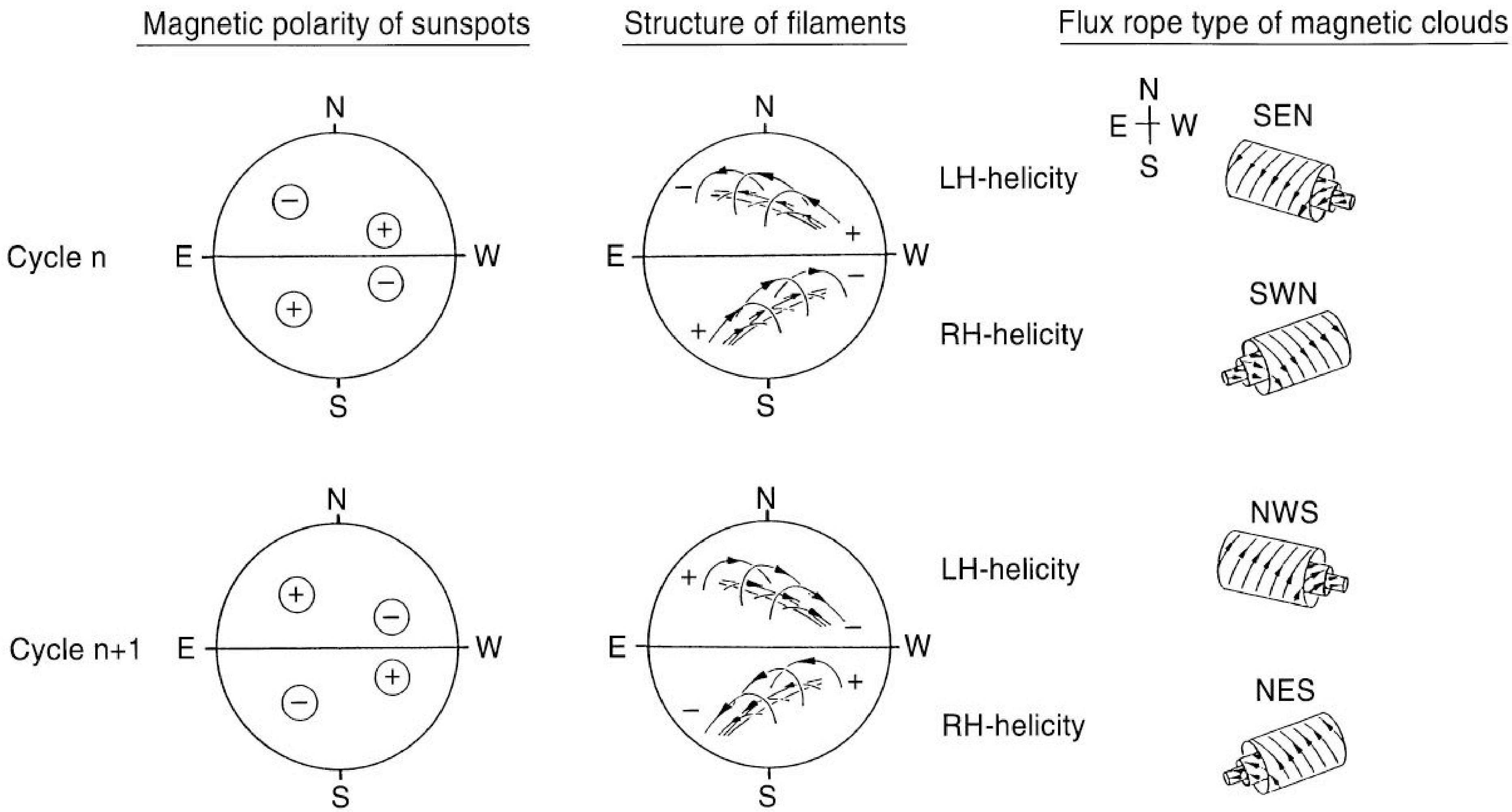
Lepping et al. [1990]



Lepping et al. [1997]

Motivation: Solar Sources of CMEs

- Magnetic coherence leads one to associate in-situ fields with solar sources. Overall trends emerge, solar-cycle dependences, AR-ICME links, etc.



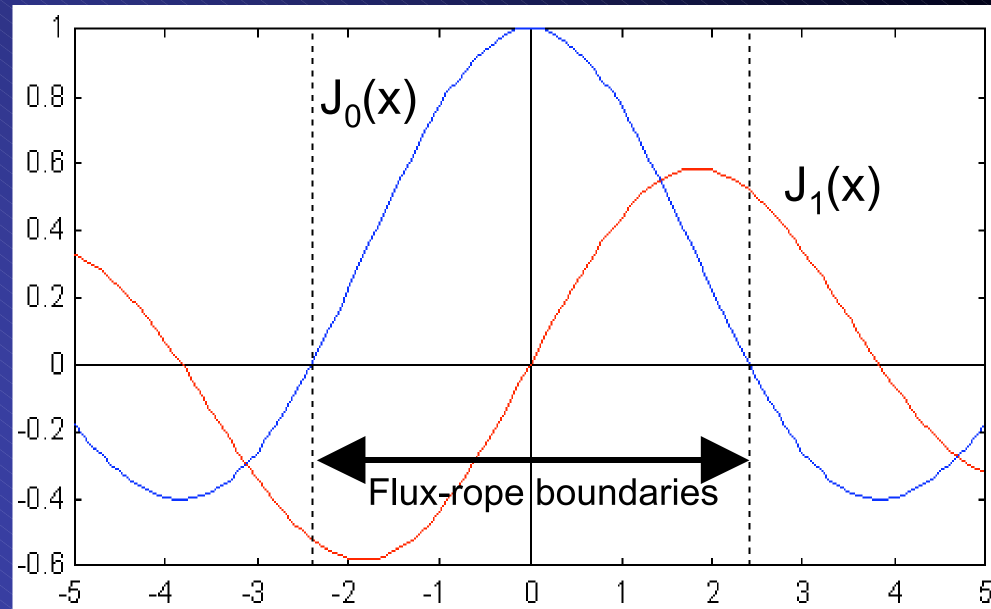
Bothmer and Rust [1997]

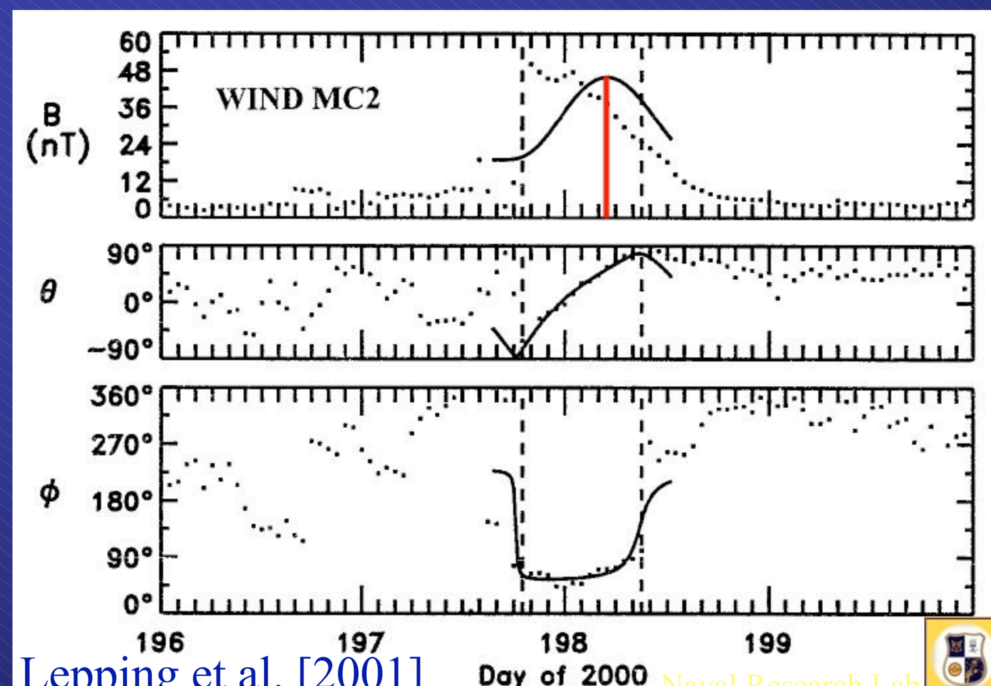
Lundquist Solution

- Linear, force-free: $\nabla \times \mathbf{B} = \alpha \mathbf{B}$
- Solution in cylindrical coordinates:
  $\mathbf{B} = HB_0 J_1(\alpha r)\,\hat{\phi} + B_0 J_0(\alpha r)\,\hat{z}$
- Two parameters: B_0 and $H = \pm 1$
- Usually define outer boundary at the first zero of J_0 so that $\alpha R_c = 2.405$
- Cylinder orientation requires three spatial parameters: θ_0, ϕ_0, and ψ_0.

Our version: 5 free parameters.

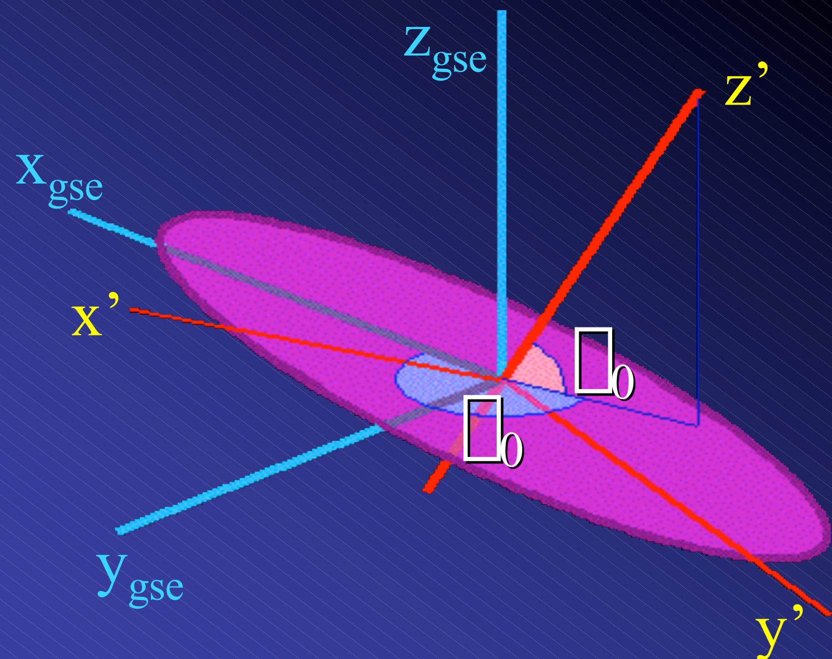
- Lepping et al. [1990] have two more parameters, t_0 and R_c for 7 total. This allows for an asymmetry solution and direct control over the size of the event.





Lepping et al. [2001]

Lundquist Solution



- Cylinder axis direction (axis of symmetry), $\hat{z}'$, defined by longitude ϕ_0, latitude θ_0

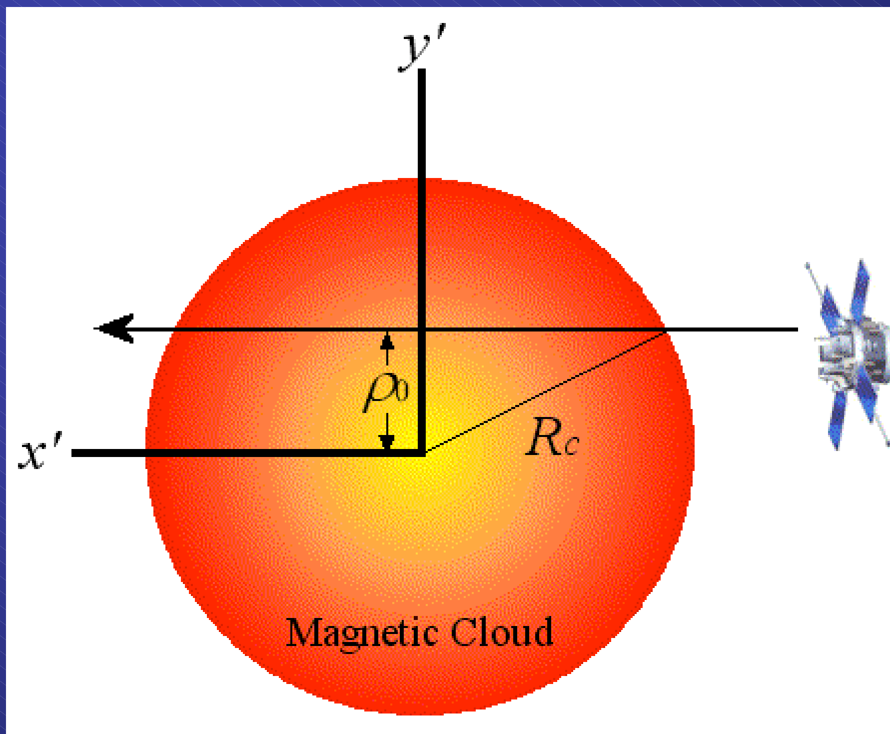
- Impact parameter ρ_0 is the closest point of approach in the cloud frame

- Spacecraft trajectory in cloud frame given by ($x'(t)$, $\rho_0 Rc$, 0), with

$$x'(t) = \langle V_r \rangle t (\hat{r} \cdot \hat{x}')$$

- Assuming a static cylinder moving at a constant speed $\langle V_r \rangle$, the cylinder radius is given by

$$R_c = \frac{\Delta T \langle V_r \rangle}{2\sqrt{1-\rho_0^2}} \sqrt{\sin^2\theta_0 + \cos^2\theta_0 \sin^2\phi_0}.$$



Fitting Procedure

1. Lepping et al. and others use MVA analysis for first guess at cylinder axis orientation $\hat{z}_o$,

$$\chi^2_{\mathrm{MVA}} = \frac{1}{N} \sum \left(\vec{B} \cdot \hat{z}_0 - \langle \vec{B} \rangle \cdot \hat{z}_0 \right)^2.$$

2. Optimize the fit parameters through 2-step minimization of directional and magnitude error norms. Fit $\square_0$, $\square_0$, and $\square_0$ with

$$\chi^2_{\mathrm{dir}} = \frac{1}{N} \sum \left(\tilde{B}_x - \tilde{B}_x^M \right)^2 + \left(\tilde{B}_y - \tilde{B}_y^M \right)^2 + \left(\tilde{B}_z - \tilde{B}_z^M \right)^2,$$

where $\tilde{B}_i = B_i / \max[|B|]$.

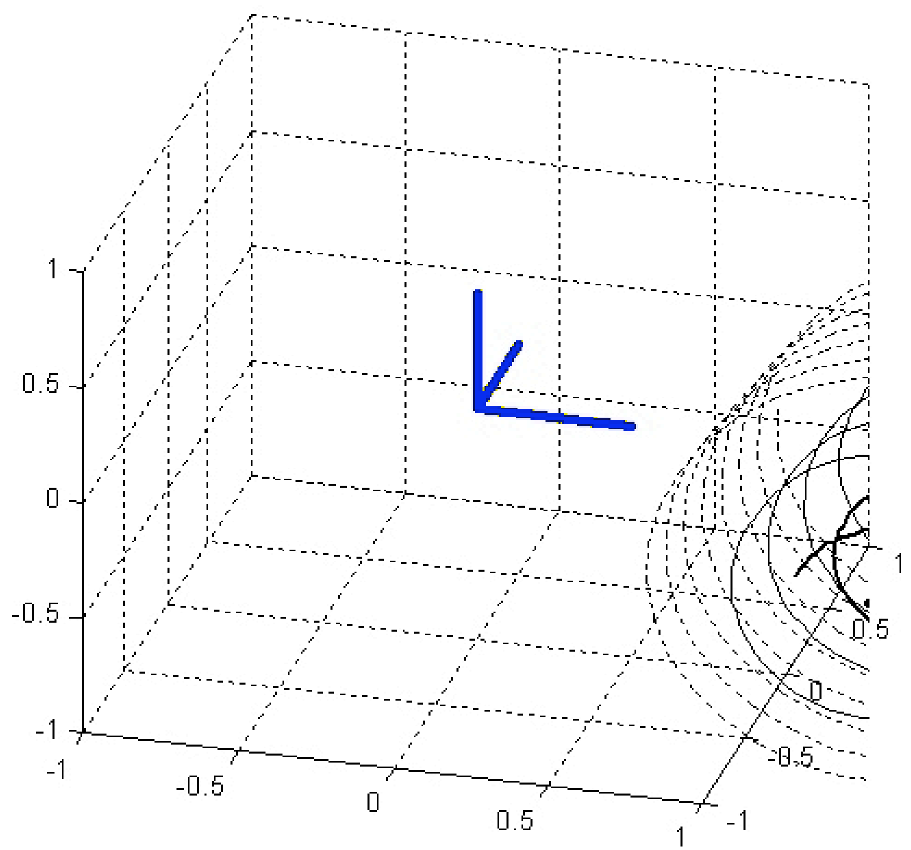
Next, fit the axial magnetic field strength B_0 with

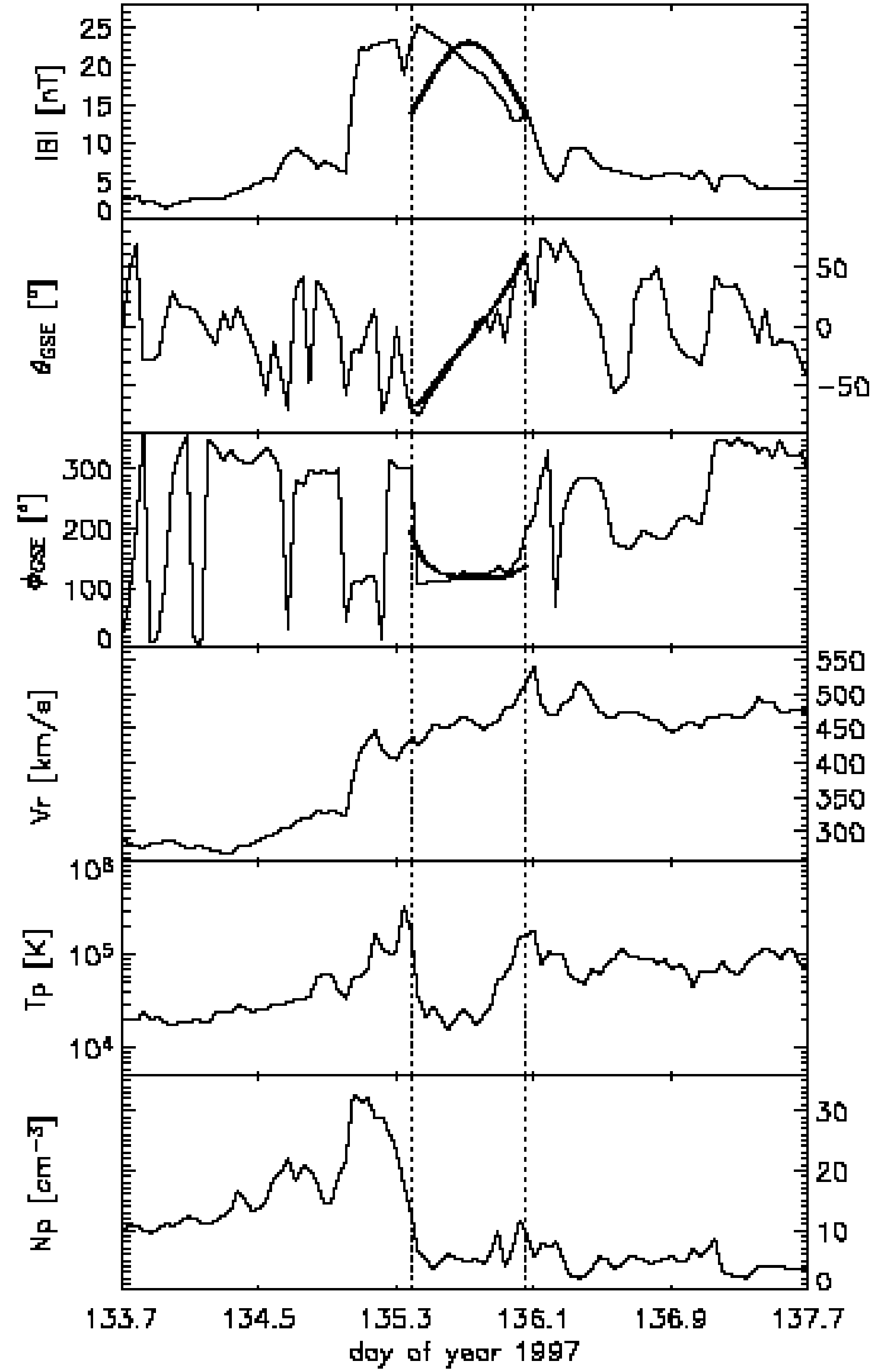
$$\chi^2_{\mathrm{mag}} = \frac{1}{N} \sum \left(|B| - |B^M| \right)^2.$$

Example Fits

□$_0$ = 92.6°
□$_0$ = -12.7°
□$_0$ = -0.35
H = -1 (LH)
B_0 = 25.2 nT

$\langle V_r \rangle$ = 466.2 km/s
□T = 17 hr
R_c = 0.096 AU
X^2_{dir} = 0.091 (good)
X^2_{mag} = 22.95





Example Fits

$□_0$ = 348.7°

$□_0$ = -22.3°

$□_0$ = 0.0

H = +1 (RH)

B_0 = 11.9 nT

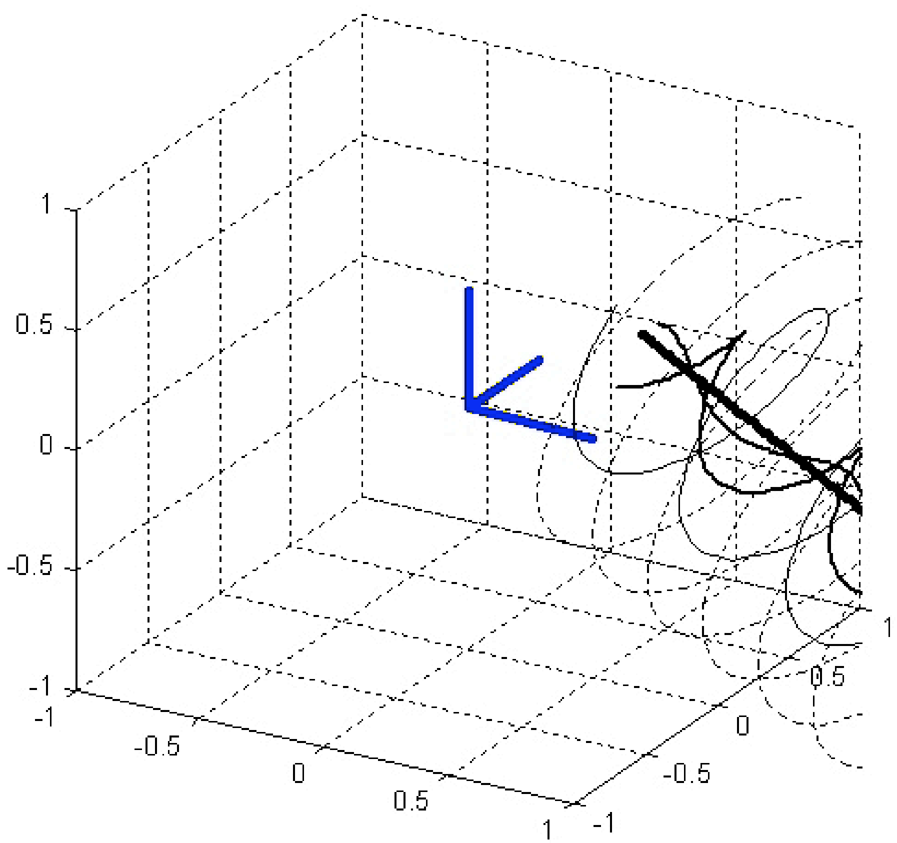
$\langle V_r \rangle$ = ~550 km/s

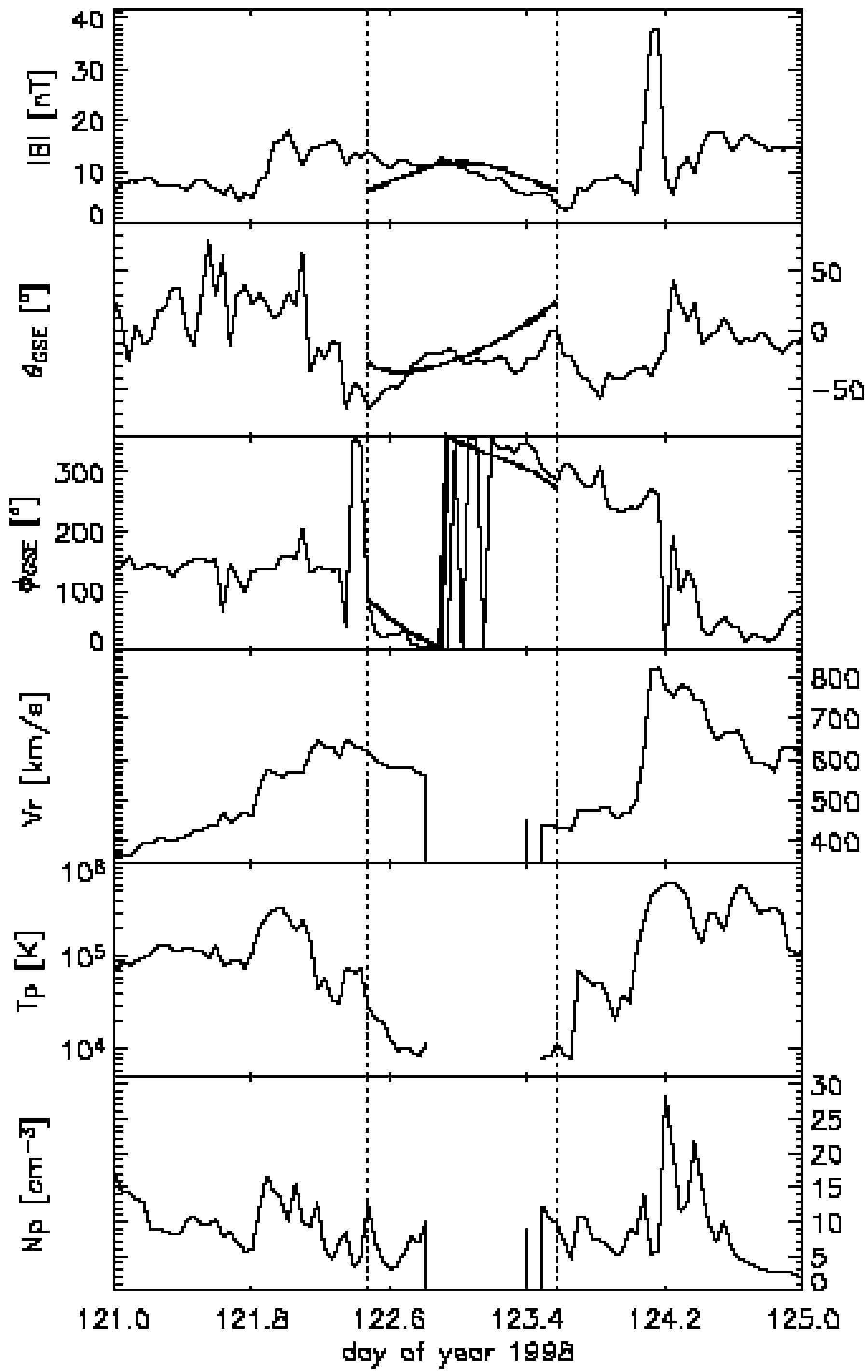
□T = 27 hr

R_c = 0.072 AU

X^2_{dir} = 0.166 (mod)

X^2_{mag} = 8.63





Fitting Uncertainties

1. Correlation between fit parameters; best-fit parameters are not necessarily unique. From 56 MC events in [Lynch et al. 2003], averaged elements of the (normalized) covariance matrix during minimization:

$$\langle |r_{\square\square}| \rangle = 0.45 \pm 0.34 \qquad \langle |r_{\square\square}| \rangle = 0.79 \pm 0.31 \qquad \langle |r_{\square\square}| \rangle = 0.63 \pm 0.35$$

$$\langle |r_{B\square}| \rangle = \langle |r_{B\square}| \rangle = 0 \qquad \langle |r_{B\square}| \rangle = 0.15 \pm 0.15$$

2. Estimates of the standard errors of best-fit parameters

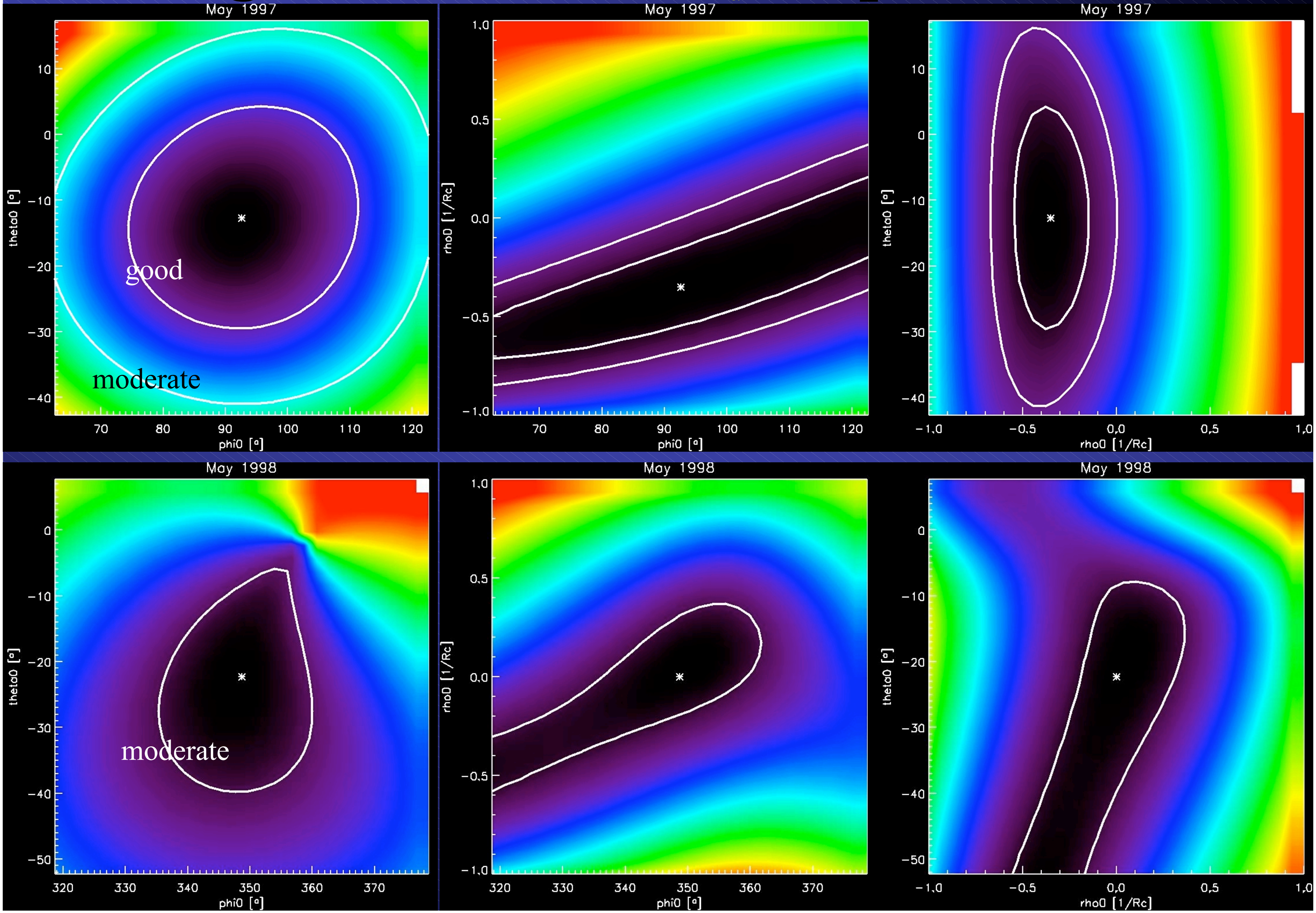
   Lepping et al. [2003] do Monte Carlo simulations of trend noise. Finds 1-□ uncertanties are functions of fit-parameter values, input noise level, etc. Lots of tables, formulae

$$\square\square_0 \sim 20 \square 30^\circ, \qquad \square\square_0 \sim 10 \square 20^\circ, \qquad \square\square_0 \sim 0.30, \qquad \square B_0 \sim 10\%$$

   Lynch et al. [2004] use fitting covariance matrix values from ~100 events to find average fitting uncertainties of:

$$\langle \square\square_0 \rangle = 37^\circ, \qquad \langle \square\square_0 \rangle = 22^\circ, \qquad \langle \square\square_0 \rangle = 0.24$$

Fitting Uncertainties : χ^2_{dir} maps



Fitting Uncertainties

Flux (axial)

$$\Phi_t = \int \mathbf{B} \cdot d\mathbf{a} = \frac{2\pi J_1(x_{01})}{x_{01}} B_0 R_c^2$$

$$\frac{\delta\Phi_t}{\Phi_t} = \sqrt{\left(\frac{\delta B_0}{B_0}\right)^2 + \left(2\frac{\delta R_c}{R_c}\right)^2},$$

Helicity

$$K = \frac{1}{\pi}\int_V B^2 dV = \frac{x_{01}}{2\pi} L B_0^2 R_c^3$$

$$\frac{\delta K}{K} = \sqrt{\left(2\frac{\delta B_0}{B_0}\right)^2 + \left(3\frac{\delta R_c}{R_c}\right)^2},$$

(For me) Uncertainty in R_c (dominated by δp_0) is the problem…

$$R_c = \frac{\Delta T \langle V_r \rangle}{2\sqrt{1 - p_0^2}} \sqrt{\sin^2\phi_0 + \cos^2\phi_0 \sin^2\theta_0}.$$

$$\left(\frac{\delta R_c}{R_c}\right)^2 = \left(\frac{\partial R_c}{\partial p_0}\frac{\delta p_0}{R_c}\right)^2 + \left(\frac{\partial R_c}{\partial \phi_0}\frac{\delta \phi_0}{R_c}\right)^2 + \left(\frac{\partial R_c}{\partial \theta_0}\frac{\delta \theta_0}{R_c}\right)^2 + \left(\frac{\partial R_c}{\partial \Delta T}\frac{\delta \Delta T}{R_c}\right)^2 + \left(\frac{\partial R_c}{\partial \langle V_r \rangle}\frac{\delta \langle V_r \rangle}{R_c}\right)^2.$$

Helicity density: $k = \dfrac{K}{V} \sim \dfrac{L B_0^2 R_c^3}{L \pi R_c^2} \sim B_0^2 R_c$

Conclusions

- LFF static cylinder a useful but simple model
- Model & fitting procedure do a reasonable job describing "classic" MC events, large rotations (~180^o), etc.
- Less reasonable for events with large $\square_0$, or less "classic" signatures (smaller rotations, < ~90^o)
- Model has its weaknesses! Best-fit parameters are often a little fuzzy. Still good for overall structural estimates, but precision is only 10s of degrees
- Flux + Helicity errors a little bit problematic. Helicity density may be the way to go…
- Didn't even mention boundary selection…