Statistical Acceleration in the Solar Corona and the Solar Wind

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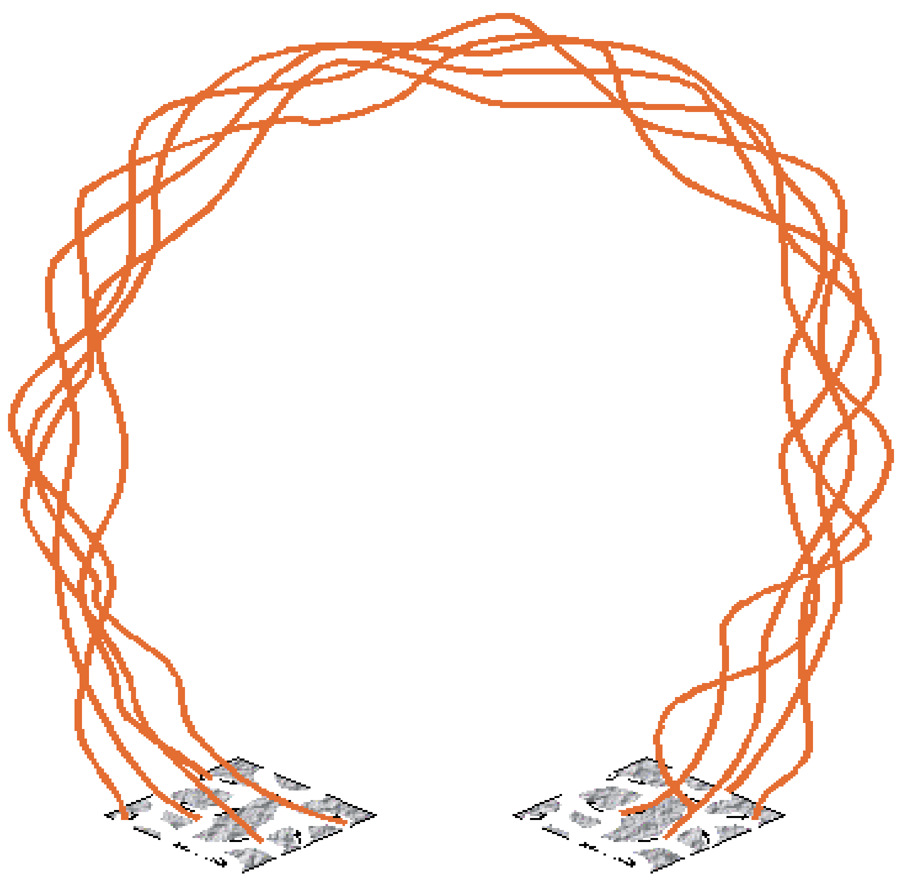
The Basic Mechanism

- There are many different contexts in which we can assume that magnetic fields are diffusing:
  - Random convective motions on the solar surface
  - Reconnection of open field lines with coronal loops and subsequent random displacements.
  - Braiding and twisting of open magnetic field lines in the overlying corona.
  - Braiding and twisting of field lines in large coronal loops.
  - Diffusion of the magnetic field, driven by speed variations in the solar wind.

Large-Coronal Loop



The Equation that Describes the Diffusion of the Magnetic Field

- The equation that describes the diffusion of the magnetic field **B** is:

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \nabla \times (\kappa \mathbf{B}) + \nabla \times (\mathbf{u}_o \times \mathbf{B})$$

- Here, κ is the diffusion coefficient for random motions of the magnetic field; $\mathbf{u}_o$ is the mean flow velocity of the plasma.
- Reduces to normal diffusion equation through vector identities.

The Equation that Describes the Diffusion of the Magnetic Field

- Consider case where $\nabla \times \mathbf{B}$ is normal to $\mathbf{B}$ and rearrange:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\left(\mathbf{u}_\kappa + \mathbf{u}_o \right) \times \mathbf{B} \right)$$

- where

$$\mathbf{u}_\kappa = \frac{\nabla \times \left(\kappa \mathbf{B} \right) \times \mathbf{B}}{B^2}$$

- $\mathbf{u}_\kappa$ is analogous to the effective mean flow velocity in energetic particles diffusion: $u_{eff} = -\kappa_{part} \nabla n / n$

Using the Tools of Energetic Particle Transport

- The mean convective velocity of energetic particles is $\mathbf{u}_\kappa + \mathbf{u}_o$ and all the formalism of energetic particle transport applies, in particular the mean change in energy is:

$$D_T \equiv \frac{\Delta T}{\Delta t} = -\nabla \cdot \left(\mathbf{u}_\kappa + \mathbf{u}_o \right) \frac{2T}{3}$$

- where T is kinetic energy; we assume particles are non-relativistic.

- $\nabla \cdot \mathbf{u}_o$ gives adiabatic cooling/heating; if $\nabla \cdot \mathbf{u}_\kappa < 0$, we will have an acceleration.

Evaluating the Acceleration

- Use following simplifications: κ varies slowly on scale of $\nabla \times \mathbf{B}$; essentially homogeneous turbulence, spatial variations of B^2 small; time stationary, i.e. replenish turbulence as damp.
- Then,

$$\nabla \cdot \mathbf{u}_\kappa = -\frac{\kappa(\nabla \times \mathbf{B})^2}{B^2}$$

- And

$$D_T = \frac{\kappa(\nabla \times \mathbf{B})^2}{B^2}\frac{2T}{3}$$

Statistical Acceleration

- We should allow for the possibility that $(\nabla \times \mathbf{B})^2$ varies with location; there is both a mean and a mean square variation in $(\nabla \times \mathbf{B})^2$, and thus a statistical acceleration is possible.
- Can derive formally, but also use trick. Note that statistical acceleration is a diffusion in momentum space, derivable from Vlasov's equation, in which case

$$D_{TT} \equiv \frac{(\Delta T)^2}{\Delta T} = 2TD_T = \frac{\kappa(\nabla \times \mathbf{B})^2}{B^2}\frac{4T^2}{3}$$

- However, to have a statistical acceleration, particles must be sufficiently mobile to sample statistically independent samples of acceleration sites.

Transport Equation

- We can write down a transport equation for the differential number density U:

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial T}\left(D_{TT} \frac{\partial U}{\partial T} \right) - \frac{\partial}{\partial T}\left(D_T U \right) - \nabla \cdot \left(\mathbf{u} U \right)$$

where, $D_T = \dfrac{2\kappa}{3B^2}(\nabla \times \mathbf{B})^2 T$; $\quad D_{TT} = \dfrac{4\kappa}{3B^2}(\nabla \times \mathbf{B})^2 T^2$; $\quad \nabla \cdot \mathbf{u} = \dfrac{\kappa}{B^2}(\nabla \times \mathbf{B})^2$

- We have ignored spatial diffusion and other convective flows

- This equation entertains power law solutions, $U \propto T^{-\beta}$, particularly when $\mathbf{u} \cdot \nabla U = 0$.
- Natural feature unlike statistical mechanisms that depend on resonance scattering.

Statistical Acceleration on a Large Coronal Loop

- Diffusion of the magnetic field within loop, driven from below.
- An issue: since diffusion is driven from below, each field line is essentially a single sample of an acceleration site. To have statistical acceleration, in which multiple acceleration sites are sampled, particles must move across field lines.
- Expect threshold energy, below which particles experience only mean acceleration; above which both mean and statistical acceleration.
- In both cases, get power law solutions:

$$U \propto T^{-\beta_1}\text{, for } T \le T_{thresh.}$$

$$U \propto T^{-\beta_2}\text{, for } T \ge T_{thresh.}$$

- And since the solutions must match at $T = T_{thresh.}$, can shown

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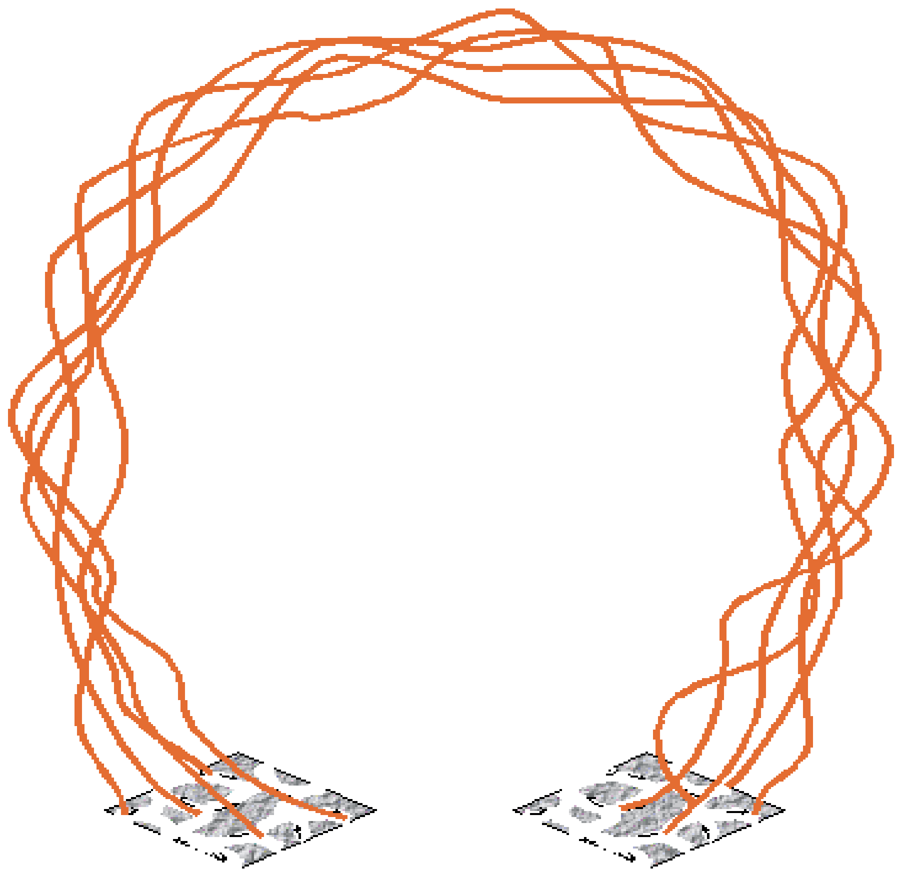
$$\beta_1 = 2\beta_2(\beta_2 - 1)$$

Large-Coronal Loop



The Threshold Energy

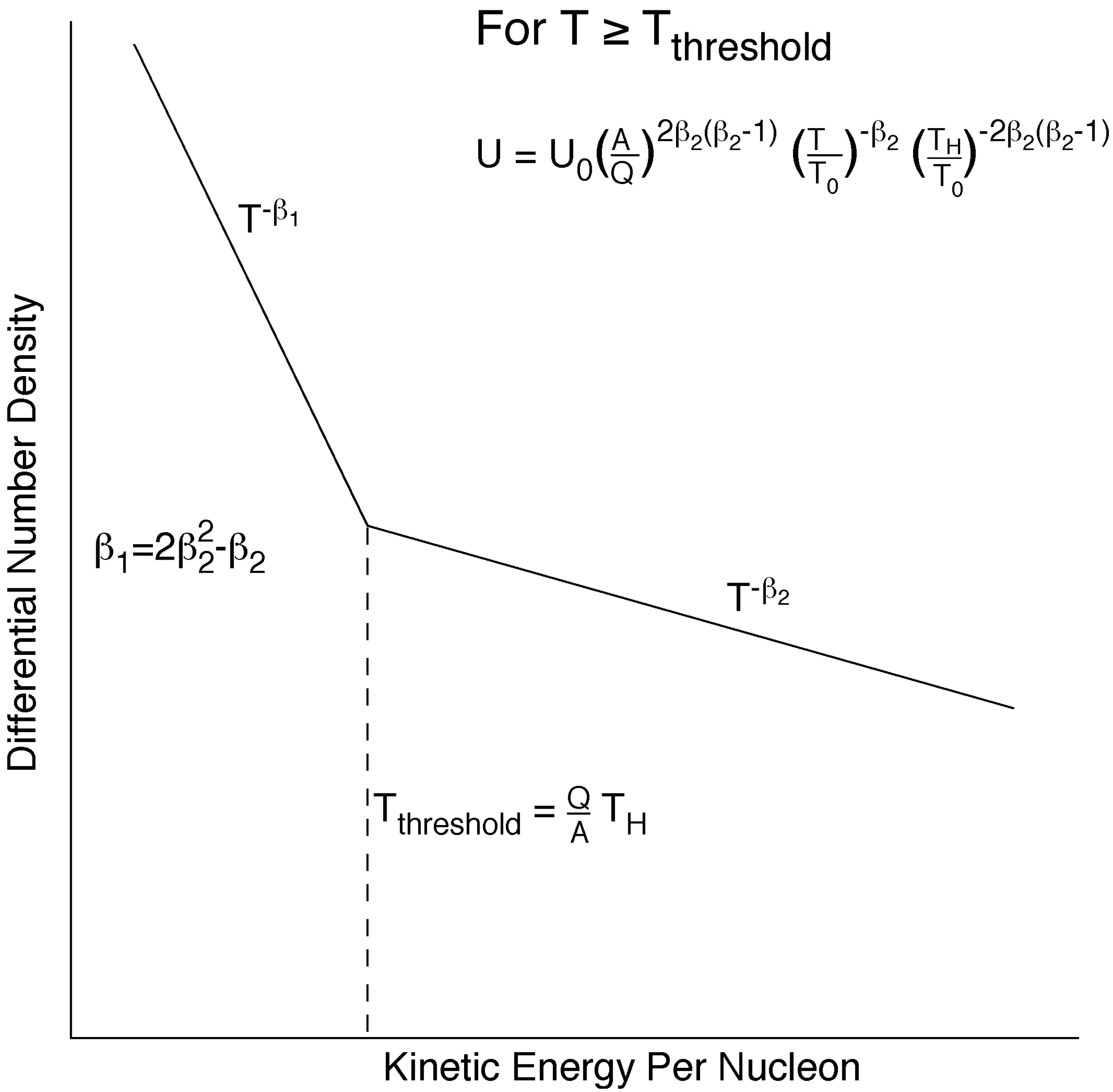
- Require particles to move across field lines to experience statistical acceleration; motions probably due to drifts.
- Convective drift velocity is, with *A* mass number; *Q*, charge: and *T* in energy per nucleon:

$$\mathbf{v}_D = \frac{2AcT}{3Q}\nabla\times\left(\frac{\mathbf{B}}{B^2}\right)$$

- Require that particles drift across field lines sufficiently rapidly within some time interval, or $\lambda / v_D <$time interval.
- Threshold energy varies as *Q*/*A*, or

$$T_{thresh} = \frac{Q}{A}T_H$$

- T_H is the threshold energy for Hydrogen.


For T ≥ T_threshold
U = U_0(A/Q)^{2β_2(β_2-1)} (T/T_0)^{-β_2} (T_H/T_0)^{-2β_2(β_2-1)}
T^{-β_1}
β_1=2β_2^2-β_2
T^{-β_2}
T_threshold = (Q/A) T_H
Differential Number Density
Kinetic Energy Per Nucleon

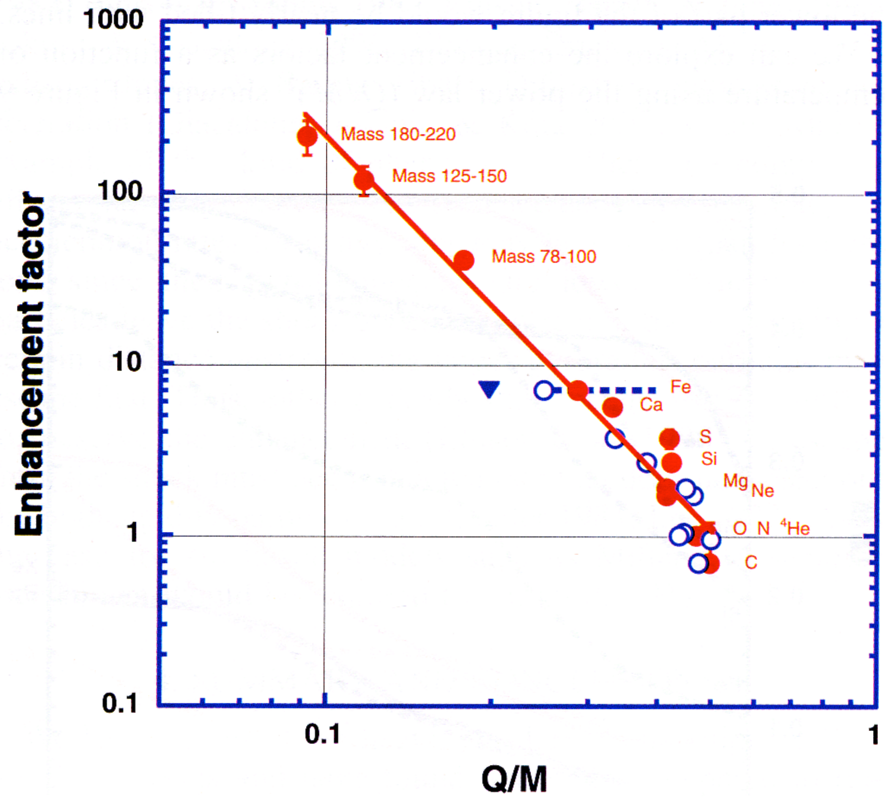


FIG. 9.—Abundance enhancement factor for impulsive-event survey vs. *Q/M* ratio, using charge state from calculations of a 3.2 MK equilibrium plasma (*red filled circles*), direct measurements of energetic particles in three large solar particle events (*blue circles*), direct measurements of Fe ionization state in the 1972 November large-particle event (*blue inverted triangle*), and a range of Fe ionization states observed in several impulsive solar particle events (*dashed line*). The solid line is a power-law fit to the red points and has a slope of −3.26.

A Relationship Between the Compositional Enhancements and Spectral Index

- We predict a relationship between the power law dependence for A/Q and spectral index
- For observed A/Q power law of ~ 3.3, $\beta_2 = 1.88$, which corresponds to differential intensity spectral index of 1.38
- Observed spectral indices at about 2 MeV/nucleon are 1.15-1.38.
- Somewhat tricky to compare since observed spectral indices are after propagation.

Some Implications

- Acceleration should occur continuously on loop; suggests impulsive flares occur when loop is disrupted and material released .
- Release of material from flares is always a problem; Reames triggers impulsive flares through reconnection with open field lines.
- Disruption might preferentially heat He-3; and cause particles to rain downward to be seen as flare.

Statistical Acceleration in Corona

- Magnetic diffusion can be argued to occur throughout the corona; use to distribute open flux and heat/accelerate the solar wind.
- May be that there is also an accompanying statistical acceleration of energetic particles in corona.
- Probably don't get compositional enhancements; particles run along field lines to see statistically independent acceleration sites.

Statistical Acceleration in Corona

- But, generate power law, superthermal tails, that result in excellent seed population for CME-driven shocks.
- May be able to use observations of superthermal tails in solar wind to tell about statistical acceleration in corona, and thus coronal heating mechanisms.
- More work needs to be done on this.