Supergranulation Waves in the Subsurface Shear Layer

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Observations

- Supergranulation pattern appears to rotate faster than plasma.
- Gizon et al. (2003) observed supergranulation waves with periods of 6-9 days and phase speeds ~65 m/s.

Gizon, L., Duvall, T. L. Jr, Schou, J. Wave-like properties of solar supergranulation. *Nature* **421**, 43-44 (2003)

Dispersion Relation:

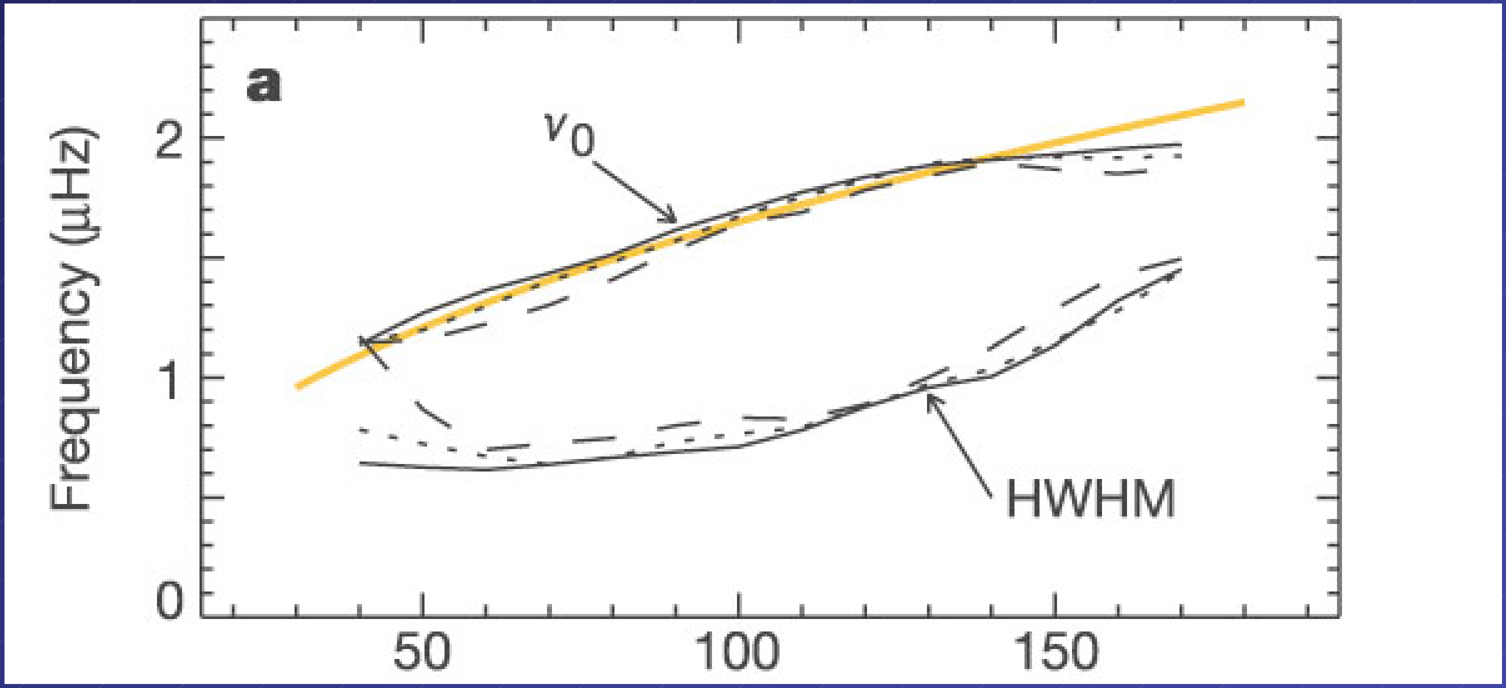


Figure from Gizon et al. (2003)

Motivation

- Want to investigate the effect of subsurface shear flow on convective modes.
- Main result is that convective modes become running waves in the presence of shear.
- Goal to calculate phase speed of convective modes and compare with observations.

Formulation

- First solve this problem in the linear approximation.
- Problem was first formulated by Adam (1977).

Adam, J. A. Hydrodynamic Instability of Convectively Unstable Atmospheres in Shear Flow. *Astrophysics and Space Science* **50**, 493-514 (1977)

Physical situation:

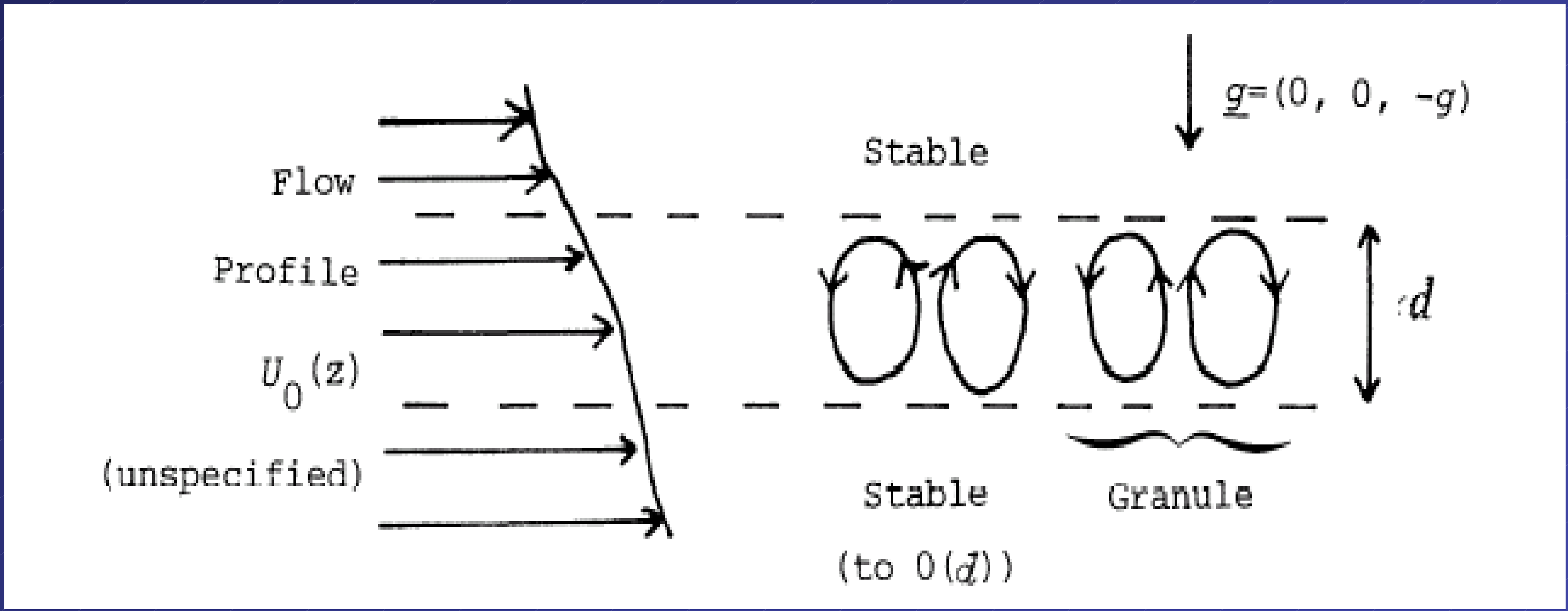


Figure from Adam (1977)

Model

Adam (1977): idealized physical situation

- ignored thermal diffusivity and viscosity
- imposed negative entropy gradient
- convectively unstable region bounded above and below by convectively stable regions

- Assume that over depth of order of unstable region, regions above and below are stable
- Reasonable for supergranular/granular interaction
- Justified in taking supergranular motion to be represented by shear flow in sub-photospheric granular convection zone

Equations

- Derived from linearized equations of continuity, motion and adiabatic compressibility.
- Equations in terms of pressure perturbation and vertical displacement.
- Both treated as periodic functions of horizontal coordinates and time.

Linearized Equations

$$\frac{d\rho_1}{dt} + \rho_0 \nabla \cdot \vec{u} + u_{1z} \frac{d\rho_0}{dz} = 0$$

$$\rho_0 \frac{du_{1x}}{dt} + \rho_0 u_{1z} \frac{dU_0}{dz} = -\frac{\partial p_1}{\partial x}$$

$$\left(\frac{d}{dt} \equiv \frac{\partial}{\partial t} + U_0 \frac{\partial}{\partial x} \right)$$

$$\rho_0 \frac{du_{1y}}{dt} = -\frac{\partial p_1}{\partial y}$$

$$\frac{dp_1}{dt} + u_{1z} \frac{dp_0}{dz} = c_0^2 \left(\frac{d\rho_1}{dt} + u_{1z} \frac{d\rho_0}{dz} \right)$$

$$\rho_0 \frac{du_{1z}}{dt} = -\frac{\partial p_1}{\partial z} - \rho_1 g$$

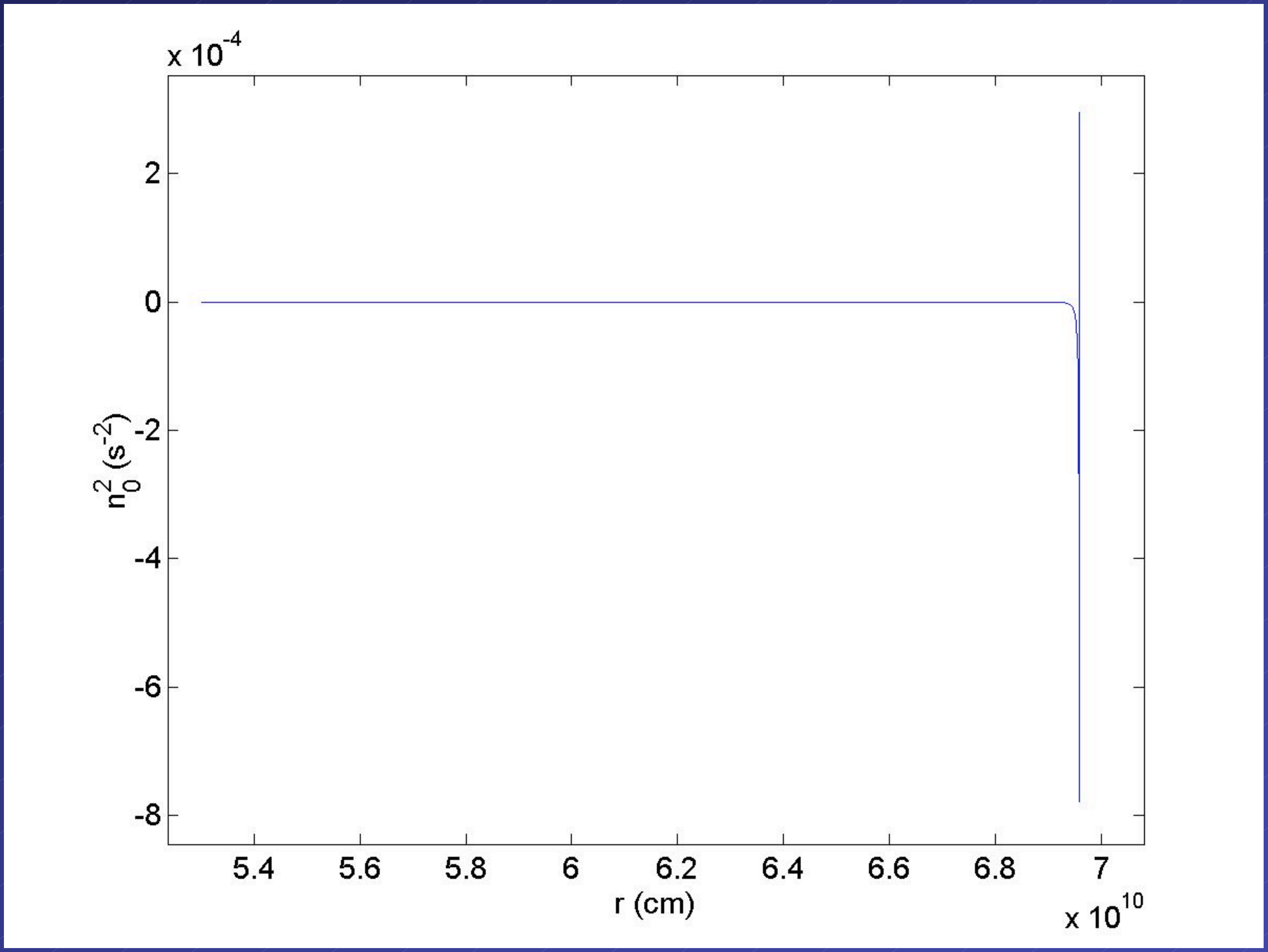
Equations

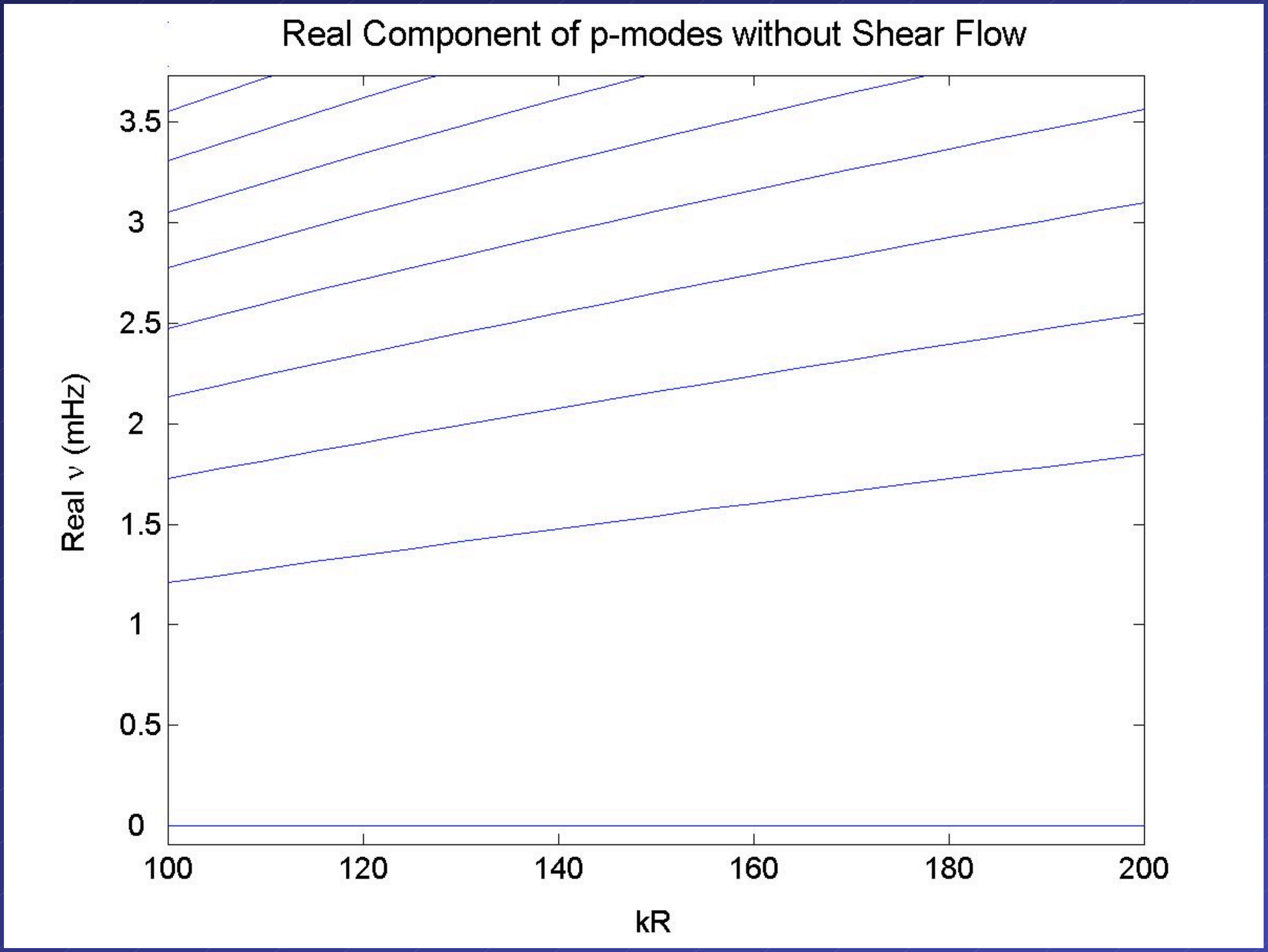
$$\left(\frac{d}{dz}+\frac{g}{c_0^2}\right)p+\tilde{n}_0\Big(-(\omega-U_0k)^2+n_0^2\Big)q=0$$

$$-(\grave{u}-U_0k)^2\left(\frac{d}{dz}-\frac{g}{c_0^2}\right)q-\tilde{n}_0^{-1}\left(-k^2+\frac{1}{c_0^2}(\grave{u}-U_0k)^2\right)p=0$$

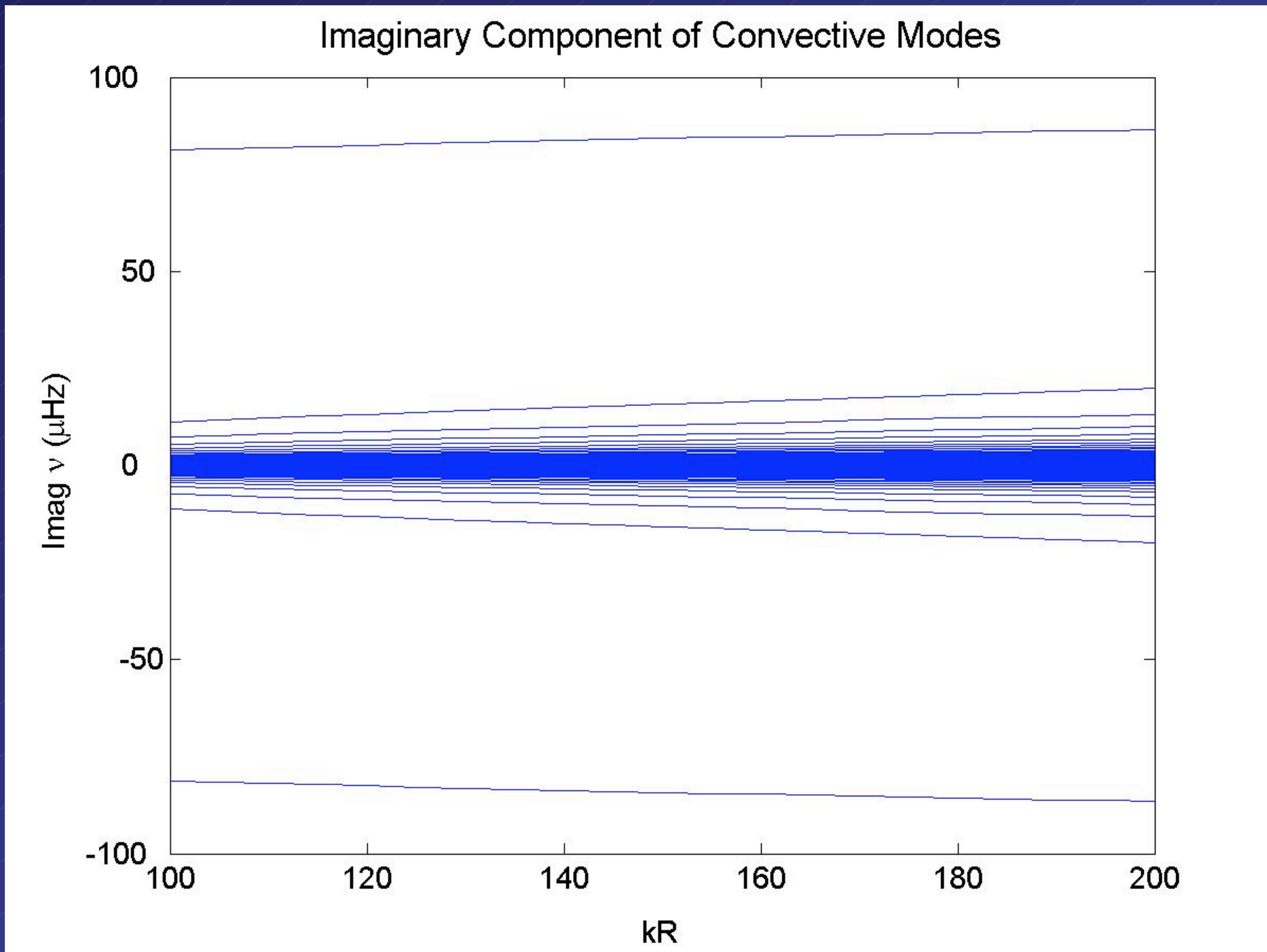
Solve for frequencies at given wavenumbers using finite differences.

Brunt-Väisälä frequency: $n_0^2 = -\frac{g^2}{c_0^2} - \frac{g}{\rho_0}\frac{d\rho_0}{dz}$

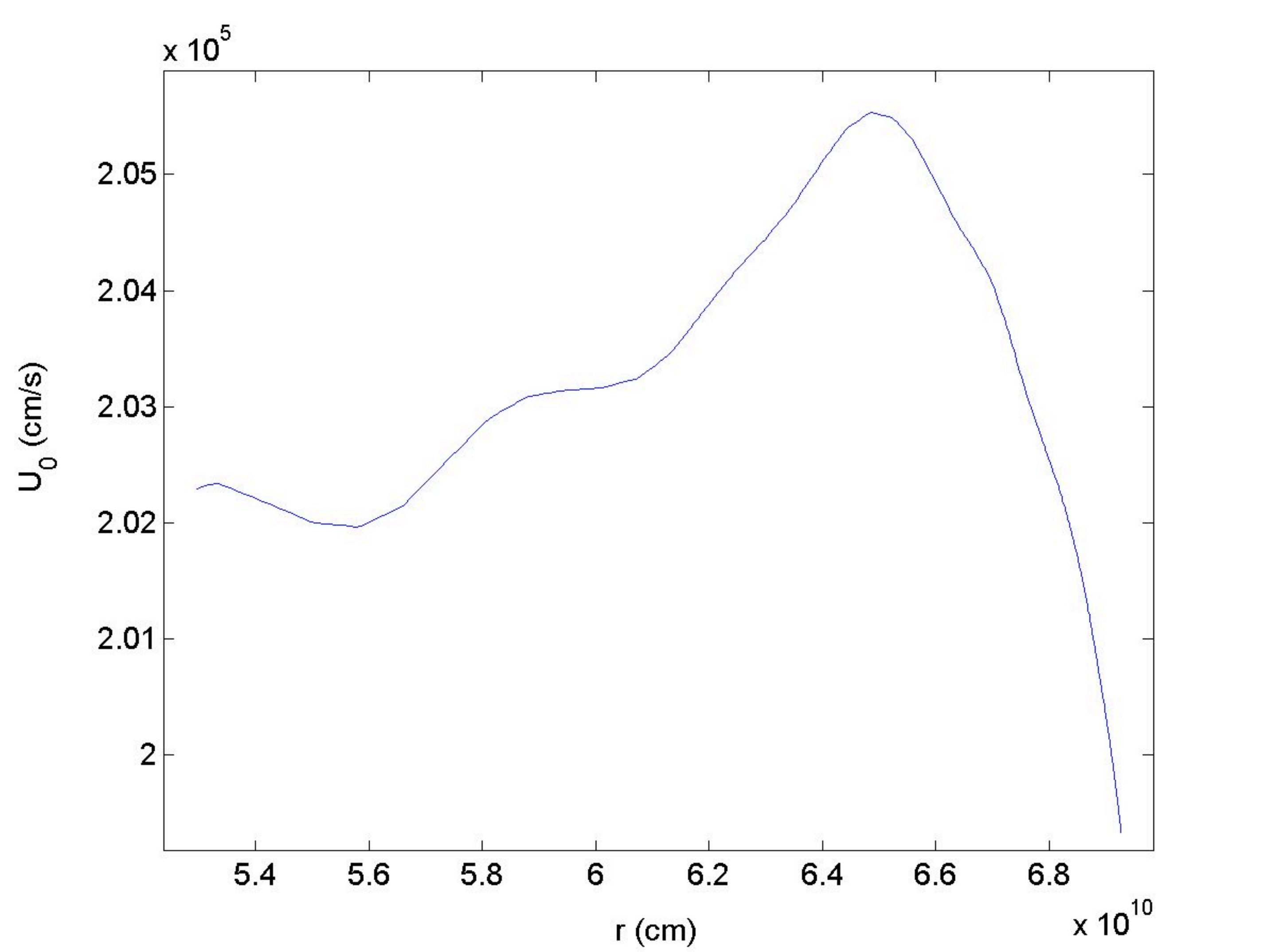


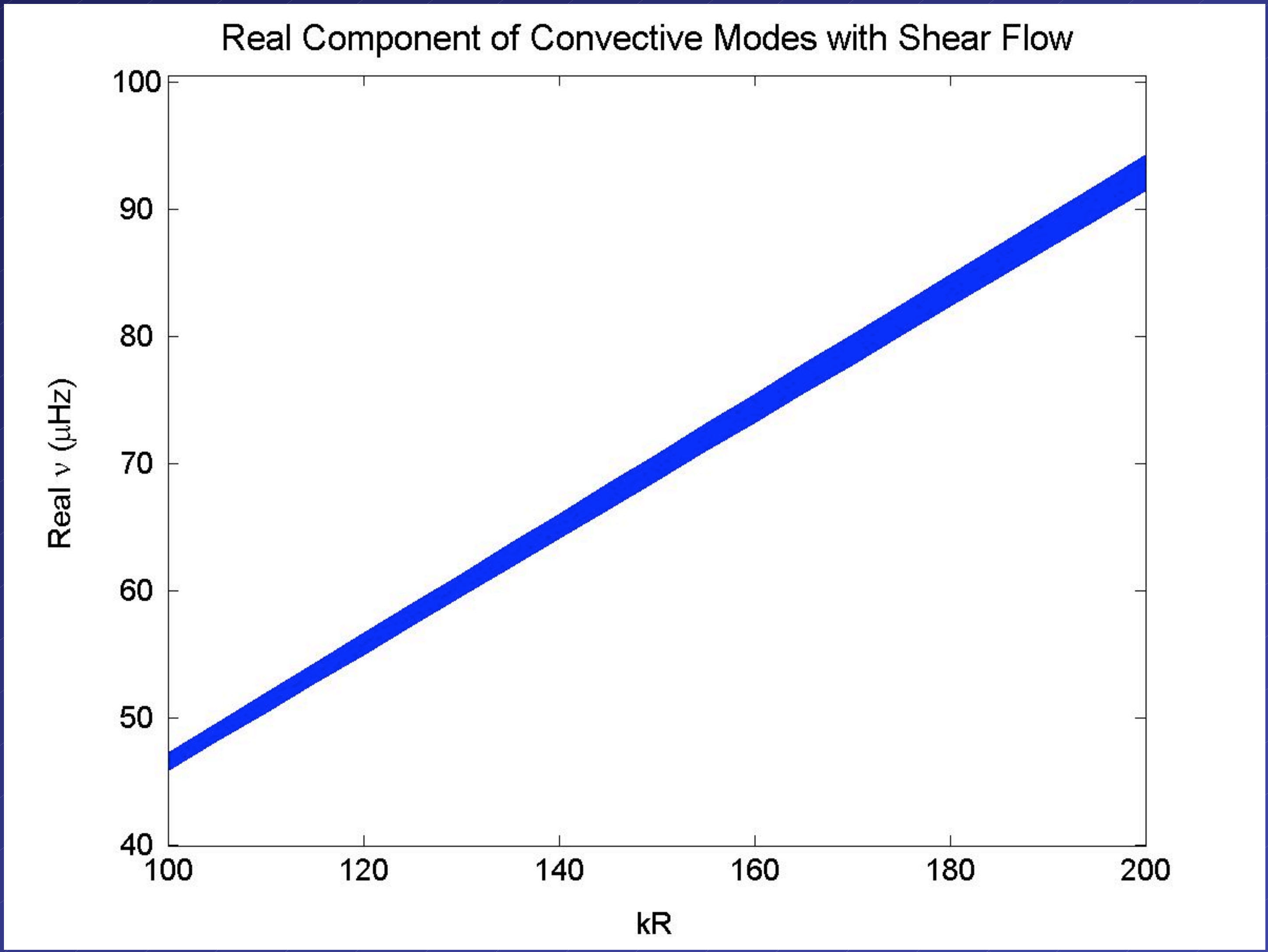


Consider range of k corresponding to typical supergranulation size.


Imaginary Component of Convective Modes
Imag ν (μHz)
100
50
0
-50
-100
100
120
140
160
180
200
kR

Solar Model Shear Flow



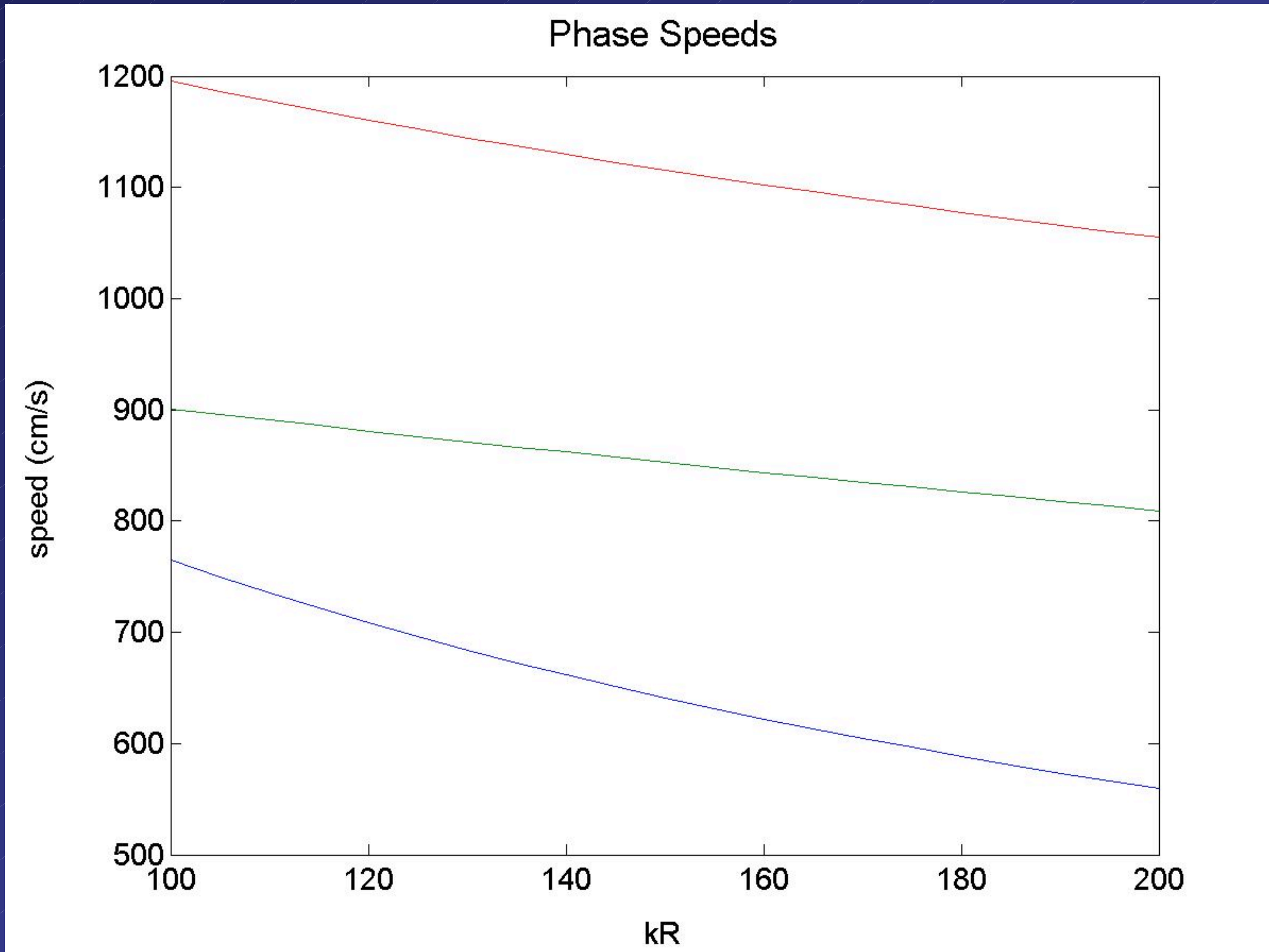


Real Component of Convective Modes with Shear Flow
100
90
80
70
60
50
40
Real ν (μHz)
100
120
140
160
180
200
kR

Phase Speeds

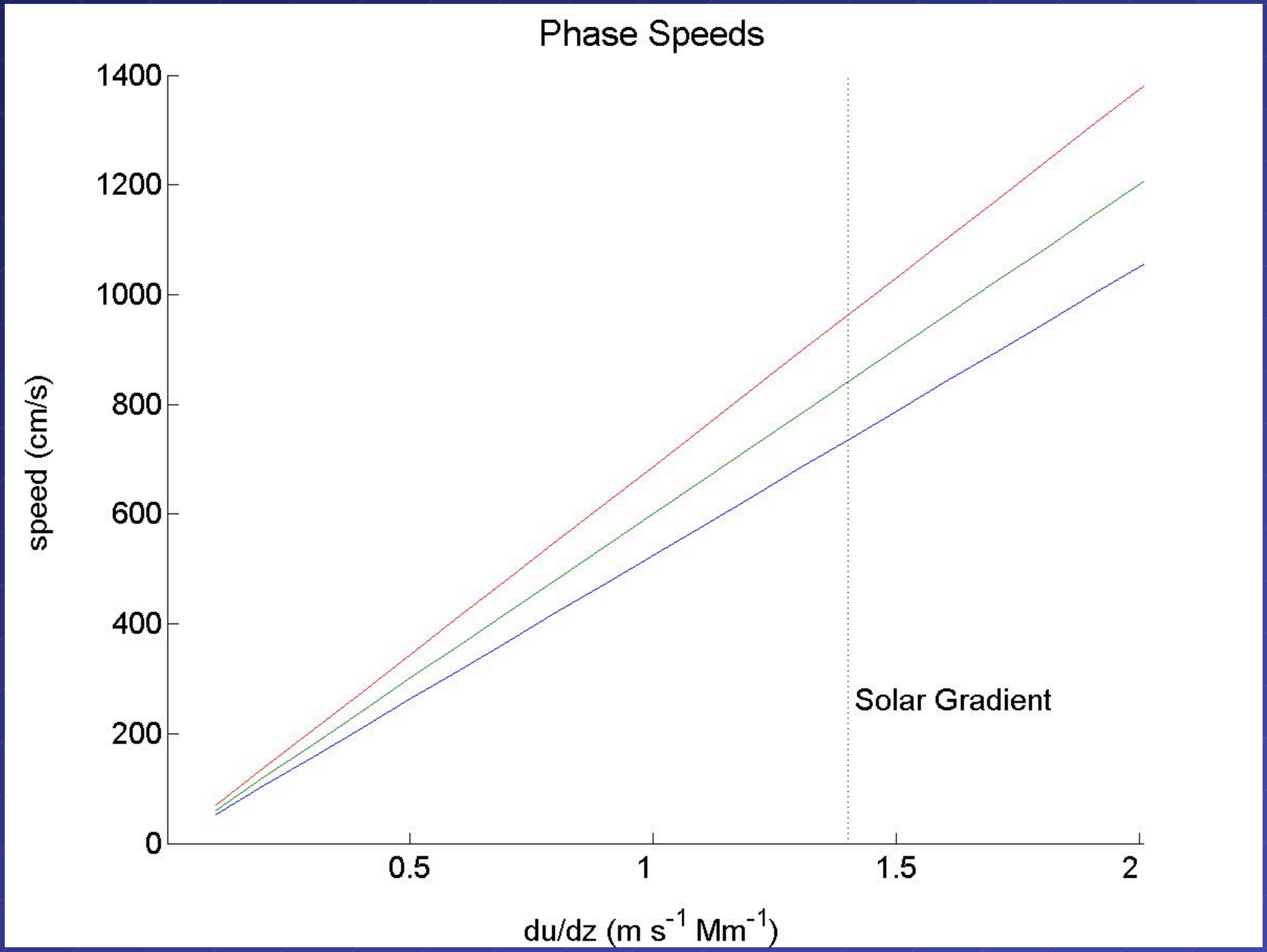
Calculate phase speeds, $$u = \frac{2\pi\upsilon}{k}$$

for first three convective modes. Subtract surface speed, to observe how much the phase speed of the convective waves exceeds the plasma speed.


Phase Speeds
1200
1100
1000
900
800
700
600
500
100
120
140
160
180
200
speed (cm/s)
kR

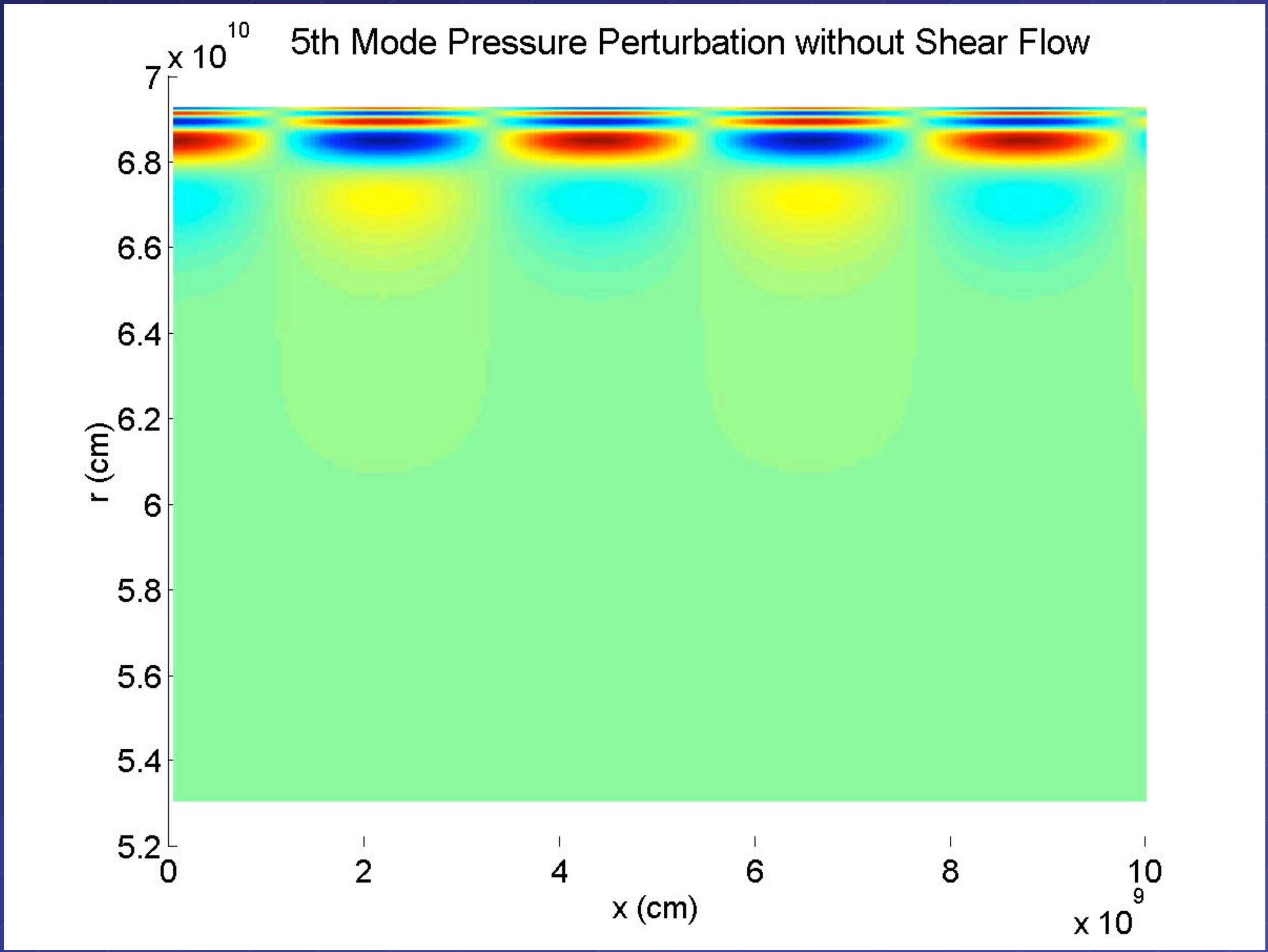
Shear Gradient

- Phase speed dependence on gradient of shear flow profile.
- Calculate with series of shear profiles, all with zero surface speed and constant gradients.

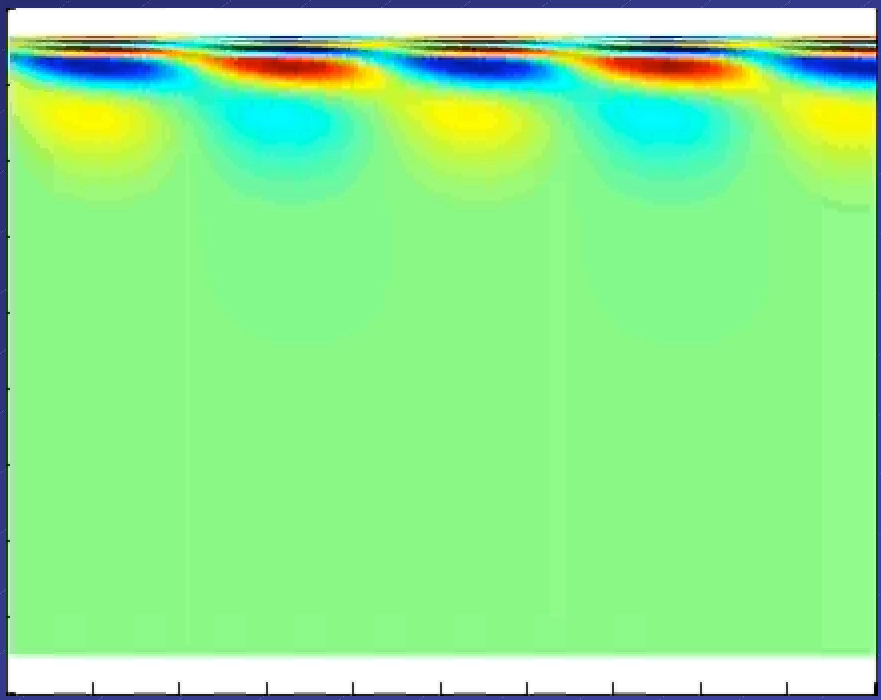


Phase Speeds
1400
1200
1000
800
600
400
200
0
speed (cm/s)
0.5
1
1.5
2
du/dz (m s^-1 Mm^-1)
Solar Gradient

Eigenfunctions



With Shear Flow:



Conclusions

- Model shows evidence of wave behaviour of supergranulation, with supergranules travelling faster than plasma.
- The wave behaviour is caused by the subsurface shear flow.
- Speeds obtained still slower than observations. Need to consider non-linear model.